EQUATIONS OF ROTATIONAL MOTION OF A GRAVITATIONAL SATELLITE EQUIPPED WITH DEFORMABLE STABILIZERS

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In connection with many problems of investigation of near-terrestrial space, interest has increased greatly recently in theoretical problems and in actual construction of satellites which are oriented toward the Earth. In this regard, objects which are oriented by means of a gravitational effect play a particularly large role. The principle of gravitational orientation, already known at the time of Lagrange, is reflected in a number of interesting papers devoted to the investigation of the conditions of existence and stability of positions of relative equilibrium of a rigid satellite in an orbital system of coordinates attached to its center of mass [1 to 3].

It is known, that the condition of optimal realization of a gravitational stabilizing system leads to the requirement that the mass of the satellite be distributed so, that in the position of equailibrium, in orbital coordinates the maximum moment of inertia lies along the binormal to the orbit [1 and 2]. The stabilizing effect may be enhanced by the use of stabilizers in the form of long bars with masses at the ends [3].

At present, long flexible stabilizers are already widely used in some satellites. These stabilizers are formed by unrolling of the treated, prestressed metal ribbons, which in their operating state have the form of lap jointed tubes. In view of the possible deformability of the stabilizers, the need arises for investigation of the effects of the dynamic phenomena, which accompany the deformation of the rods, on the spatial orientation of the satellite.

Consideration of the effects of deformation may be specially important if, in addition, use of some active system of damping and guidance abroad the satellite is contemplated.

A system of equations is presented describing the rotational motion about its mass center of a gravitational satellite, provided with deformable stabilizers.

The consideration of the dynamic phenomena which accompany the deformation of the stabilizers is carried out by the methods of analytical dynamics.

1. Equations of the theory of relative motion applicable to the dynamics of a deformable body. The equations of motion of the structure of a satellite containing elastic elements can, in the majority of practically important cases, be obtained on the basis of the general equations of the theory of relative motion. The structure may be regarded as a system of bodies whose position relative to some selected system of axes connected to an absolutely rigid body may be specified by a finite number of generalized coordinates (or, for a continuous medium, by a denumerable set of them) ([4], pp. 426-436).

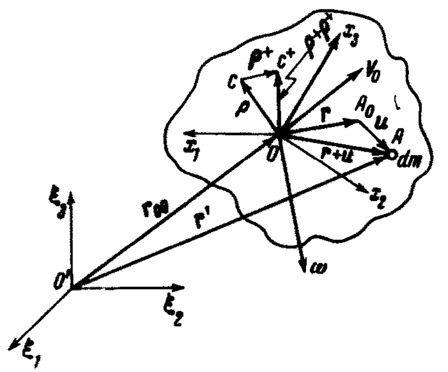

FIG. 1

We shall attach to some point O of the structure in its undeformed state a right handed coordinate system $x_1 x_2 x_3$ which pertains to this body, and we shall describe its motion in terms of the velocity of the origin $\mathbf{V}_0$ and its angular velocity vector ω (Fig. 1). The displacements of the points of the deformable body will be determined with respect to some specified moving system of axes $Ox_1 x_2 x_3$. At any instant of time, the position of an arbitrary point A of the body relative to the $Ox_1 x_2 x_3$ system of axes, can be defined by the vector ([4], pp. 474-482)

$$OA = \mathbf{r} + \mathbf{u} \tag{1.1}$$

Here $\mathbf{r}$ is the vector for the point A, constant in the $Ox_1 x_2 x_3$ system, and $\mathbf{u}$ accounts for the displacement of an arbitrary point from its initial position A_0 to the position A under consideration, resulting from the deformation of the body only. The vector $\mathbf{u}$ is, in general, a function of time and of the coordinates of the point A. It can be assumed that time enters $\mathbf{u}$ only through the parameters which serve as generalized coordinates, the number of which is assumed to be finite and equal to N, i.e.

The position of the arbitrary point A of the body with respect to the inertial coordinate system $O' \xi_1 \xi_2 \xi_3$ can now be given by the radius vector

$$\boldsymbol{r}' = \boldsymbol{r}_{00} + \boldsymbol{r} + \boldsymbol{u} \tag{1.2}$$

If the translational motion of the body is given, then $\mathbf{r}_{00}$ will be a known function of time; otherwise, if the motion of the body is to be determined, $\mathbf{r}_{00}$ can also be determined by generalized coordinates which specify the motion of the origin.

Expressions for the absolute velocity and acceleration of the given point of the deformable body, can be given in the form

$$\mathbf{V} = \mathbf{V}_0 + \boldsymbol{\omega} \times (\mathbf{r} + \mathbf{u}) + \mathbf{u}^*$$

$$\begin{aligned} \mathbf{W} = \mathbf{V}_0{}^* + \boldsymbol{\omega} \times \mathbf{V}_0 + \boldsymbol{\omega}^{\cdot} \times (\mathbf{r} + \mathbf{u}) + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + \\ + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{u}) + 2\boldsymbol{\omega} \times \mathbf{u}^* + \mathbf{u}^{**} \end{aligned} \tag{1.3}$$

where u^*, and u^{**} are the relative velocity and acceleration vectors of the point in the $Ox_1x_2x_3$ system due to deformation, and $\mathbf{V}_0^*$ is the vector whose projections on the $Ox_1x_2x_3$ axes are equal to the derivatives of the projections of the vector $\mathbf{V}_0$ on the same axes.

The equations of motion of the deformable structure of the satellite in space can be obtained either on the basis of the theorems on the rate of change of momentum and on the rate of change of angular momentum of the structure taken about O, or as the Euler-Lagrange equations for the quasi-velocities $(V_{01}, V_{02}, V_{03}, \omega_1, \omega_2, \omega_3)$, which define both, the motion of the point O of the body, and its rotation about O. In vector notation, these equations have the form

$$\begin{gathered} M\,[\mathbf{V}_0^{**} + \boldsymbol{\omega}\times\mathbf{V}_0 + \boldsymbol{\omega}^{\cdot}\times\boldsymbol{\rho} + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\boldsymbol{\rho})] + \\ + M\,[\boldsymbol{\omega}^{\cdot}\times\boldsymbol{\rho}^{+} + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\boldsymbol{\rho}^{+}) + 2\boldsymbol{\omega}\times\boldsymbol{\rho}^{*+} + \boldsymbol{\rho}^{**+}] = \mathbf{F} \end{gathered} \tag{1.4}$$

$$\begin{gathered} (\Theta^{\circ} + \Theta^{\circ+})\cdot\boldsymbol{\omega}^{\cdot} + \Theta^{\cdot\circ+}\cdot\boldsymbol{\omega} + \boldsymbol{\omega}\times(\Theta^{\circ} + \Theta^{\circ+})\cdot\boldsymbol{\omega} + \boldsymbol{\Gamma}^{*} + \boldsymbol{\omega}\times\boldsymbol{\Gamma} + \\ + M\,(\boldsymbol{\rho} + \boldsymbol{\rho}^{+})\times[\mathbf{V}^{*}{}_0 + \boldsymbol{\omega}\times\mathbf{V}_0] = \mathbf{m}^{\circ} + \mathbf{M}^{\circ} \end{gathered} \tag{1.5}$$

where $\mathbf{F}$ is the vector sun of all external forces; $\mathbf{m}^{\circ}$ is the vector sum of moments of the external forces about O; $\boldsymbol{M}^{\circ}$ is the vector sum of the moments of the forces (including reactive forces) arising from the guidance systems, and

$$\Theta^{\circ} = E\int \mathbf{r}\cdot\mathbf{r}\,dm - \int \mathbf{r}\mathbf{r}\,dm \tag{1.6}$$

is the inertia tensor at the point O of the structure in its undeformed state, expressed in terms of the inertia tensor relative to the center of inertia C with the aid of the relation

$$\Theta^{\circ} = \Theta^{C} + M\,(E\boldsymbol{\rho}\cdot\boldsymbol{\rho} - \boldsymbol{\rho}\boldsymbol{\rho}) \tag{1.7}$$

where $\boldsymbol{\rho}$ is the constant in the $Ox_1x_2x_3$ system radius vector of the center of inertia C of the structure, i.e.

$$\int \mathbf{r}\,dm = M\boldsymbol{\rho}$$

where M is the mass of the entire structure.

$$\Theta^{\circ+} = 2\left[E\int \mathbf{u}\cdot\mathbf{r}\,dm - \frac{1}{2}\int(\mathbf{u}\mathbf{r} + \mathbf{r}\mathbf{u})\,dm\right] + E\int \mathbf{u}\cdot\mathbf{u}\,dm - \int \mathbf{u}\mathbf{u}\,dm \tag{1.8}$$

is the additional inertia tensor of deformation of the structure relative to the point O (here $\mathbf{uu}$, $\mathbf{ur}$ and $\mathbf{ru}$ are dyadic products of the vectors and E is the unit tensor); moreover, in (1.4) and (1.5) the following notations for vectors are used:

$$\begin{gathered} \boldsymbol{\Gamma} = \int(\mathbf{r} + \mathbf{u})\times\mathbf{u}^{*}\,dm \\ \int \mathbf{u}\,dm = M\boldsymbol{\rho}^{+}, \quad \int \mathbf{u}^{*}\,dm = M\boldsymbol{\rho}^{*+}, \quad \int \mathbf{u}^{**}\,dm = \mathrm{M}\rho^{**+} \end{gathered} \tag{1.9}$$

Here $\boldsymbol{\rho}^{+}$ is the vector which defines the displacement of the inertia center from the position corresponding to the undeformed state C to the position C^{+} which corresponds to the deformed state of the body. In all the relations given above, integration extends over all masses distributed and concentrated) of the deformable structure. The equations for the generalized coordinates q_α, which may be obtained as Lagrange's equations, have the form

$$\frac{d}{dt}\frac{\partial T_*}{\partial q_\alpha^{\cdot}} - \frac{\partial T_*}{\partial q_\alpha} = Q_\alpha - M(\mathbf{V}_0{}^* + \boldsymbol{\omega}\times\mathbf{V}_0)\cdot\frac{\partial \boldsymbol{\varrho}^+}{\partial q_\alpha} - \tag{1.10}$$

$$-\frac{1}{2}\boldsymbol{\omega}\cdot\frac{\partial\Theta^{\circ+}}{\partial q_\alpha}\cdot\boldsymbol{\omega} - \boldsymbol{\omega}^{\cdot}\cdot\frac{\partial\Gamma}{\partial q_\alpha^{\cdot}} - \boldsymbol{\omega}\cdot\left(\frac{d}{dt}\frac{\partial\Gamma}{\partial q_\alpha^{\cdot}} - \frac{\partial\Gamma}{\partial q_\alpha}\right) \qquad (\alpha = 1, 2, \ldots, N)$$

$$T_* = \frac{1}{2}\int \mathbf{u}^*\cdot\mathbf{u}^*\,dm \tag{1.11}$$

Here T_* is the kinetic energy of the relative motion of the particles of the body due to the deformations only. Q_α is a certain generalized force. The components which have been separated out in the right-hand side of Equations (1.10) represent the generalized forces corresponding to the inertia force of translational motion of the origin shifted to the center of inertia of the system, and also to the rotational, centrifugal, and Coriolis inertia forces on the body, caused by its deformation. The forces Q_α are the generalized forces of all the external forces (gravitational, aerodynamic, etc.) and of the internal reactions (elastic and inelastic) arising during the deformation.

It would be possible to use Equations (1.5) and (1.10) directly in the following. However, when the vector $\mathbf{u}$ can be represented by a power series in the generalized coordinates q_α, i.e.

$$\mathbf{u} = \sum_{\alpha=1}^{N} q_\alpha \mathbf{U}^\alpha(x_1, x_2, x_3) + \frac{1}{2}\sum_{\alpha=1}^{n}\sum_{\beta=1}^{n} q_\alpha q_\beta \mathbf{U}^{\alpha\beta}(x_1, x_2, x_3) \tag{1.12}$$

then Equations (1.14), (1.5), and (1.10) can be transformed somewhat, as was done in [5]. We shall give now these approximate equations, which were obtained under the assumption that only small vibrations of the structure were being considered. In the equations, therefore, only linear terms in the generalized coordinates q_α were taken into account:

$$M[\mathbf{V}_0{}^{**} + \boldsymbol{\omega}\times\mathbf{V}_0 + \boldsymbol{\omega}^{\cdot}\times\boldsymbol{\rho} + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\boldsymbol{\rho})] = \tag{1.13}$$

$$= \mathbf{F} - \sum_{\alpha=1}^{N}\{q_\alpha[\boldsymbol{\omega}^{\cdot}\times\mathbf{a}^\alpha + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{a}^\alpha)] + 2q_\alpha^{\cdot}\boldsymbol{\omega}\times\mathbf{a}^\alpha + \mathbf{a}^\alpha q_\alpha^{\cdot\cdot}\}$$

$$\Theta^\circ\cdot\boldsymbol{\omega}^{\cdot} + \boldsymbol{\omega}\times\Theta^\circ\cdot\boldsymbol{\omega} + M\boldsymbol{\rho}\times[\mathbf{V}_0{}^* + \boldsymbol{\omega}\times\mathbf{V}_0] = \tag{1.14}$$

$$= \mathbf{m}^\circ + \mathbf{M}^\circ - \sum_{\alpha=1}^{N} q_\alpha[2(\Lambda^\alpha\cdot\boldsymbol{\omega}^\alpha + \boldsymbol{\omega}\times\Lambda^\alpha\cdot\boldsymbol{\omega}) + \mathbf{a}^\alpha\times(\mathbf{V}_0{}^* + \boldsymbol{\omega}\times\mathbf{V}_0)] -$$

$$-\sum_{\alpha=1}^{N} q_\alpha^{\cdot}(2\boldsymbol{\omega}\cdot\Lambda^\alpha + \boldsymbol{\omega}\times\mathbf{G}^\alpha) - \sum_{\alpha=1}^{N} q_\alpha^{\cdot\cdot}G^\alpha$$

$$\sum_{\beta=1}^{N} A^{\alpha\beta}q_\beta^{\cdot\cdot} = Q_\alpha - \left(\mathbf{a}^\alpha + \sum_{\beta=1}^{N}\mathbf{a}^{\alpha\beta}q_\beta\right)\cdot(\mathbf{V}_0{}^* + \boldsymbol{\omega}\times\mathbf{V}_0) + \tag{1.15}$$

$$+ \omega \cdot \left(\Lambda^\alpha + \sum_{\beta=1}^{N} Q^{\alpha\beta} q_\beta\right) \cdot \omega + 2\omega \cdot \sum_{\beta=1}^{N} \Gamma^{\alpha\beta} q_\beta^{\cdot} - \omega^{\cdot} \cdot \left(\mathbf{G}^\alpha + \sum_{\beta=1}^{N} \mathbf{G}^{\beta\alpha} q_\beta\right)$$
$$(\alpha = 1, 2, \ldots, N)$$

In Equations (1.13) to (1.15) the notation of [5] is used:

$$\mathbf{a}^\alpha = \int \mathbf{U}^\alpha dm, \qquad \mathbf{a}^{\alpha\beta} = \int \mathbf{U}^{\alpha\beta} dm, \qquad \mathbf{G}^\alpha = \int \mathbf{r} \times \mathbf{U}^\alpha dm$$

$$A^{\alpha\beta} = \int \mathbf{U}^\alpha \cdot \mathbf{U}^\beta dm, \quad \mathbf{G}^{\beta\alpha} = \mathbf{\Gamma}^{\beta\alpha} + \int \mathbf{r} \times \mathbf{U}^{\beta\alpha} dm, \quad \mathbf{\Gamma}^{\beta\alpha} = \int \mathbf{U}^\beta \times \mathbf{U}^\alpha dm \tag{1.16}$$

$$\Theta^{0+} = 2\sum_{\alpha=1}^{N} \Lambda^\alpha q_\alpha + \sum_{\alpha=1}^{N}\sum_{\beta=1}^{N} Q^{\alpha\beta} q_\alpha q_\beta, \qquad \Lambda^\alpha = \int [E\mathbf{r}\cdot\mathbf{U}^\alpha - {}^1\!/_2(\mathbf{r}\mathbf{U}^\alpha + \mathbf{U}^\alpha\mathbf{r})]\, dm$$

$$Q^{\alpha\beta} = \int [E\mathbf{U}^\alpha \cdot \mathbf{U}^\beta - {}^1\!/_2(\mathbf{U}^\alpha\mathbf{U}^\beta + \mathbf{U}^\beta\mathbf{U}^\alpha) + E\mathbf{r}\cdot\mathbf{U}^{\alpha\beta} - {}^1\!/_2(\mathbf{r}\mathbf{U}^{\alpha\beta} + \mathbf{U}^{\alpha\beta}\mathbf{r})]\, dm$$

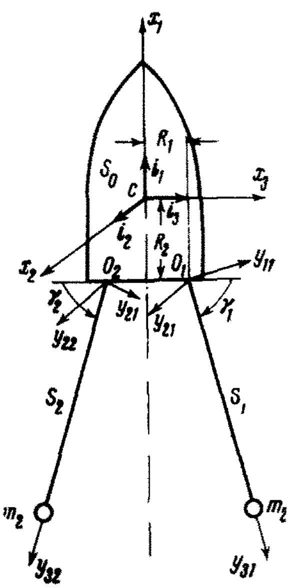

FIG. 2

2. Equations of rotational motion of a gravitational satellite with deformable stabilizers. We shall obtain the equations of rotational motion of a satellite whose structure is assumed to consist of the basic rigid body S_0 (of mass m_0) and of the stabilizers S_1 and S_2. The latter consist of long inextensible elastic bars, carrying equal concentrated masses m_2 at their ends. The bars are arranged symmetrically with respect to the x_1-axis, are rigidly attached to the basic body of the structure, and are presumed to be rectilinear in the undeformed state (Fig. 2).

The results of launchings of some satellites on which long bars were used indicate that, as a result of solar heating the stabilizers, after unrolling, will take the shape of bent and twisted beams of open section, whose axes are three-dimensional curves. However, it is hoped that in a short time an engineering solution will be found to the problem of minimizing the deflections of the axes of the bars in their operating condition. Therefore, in what follows we shall limit our consideration to small deflections of the bars (small in comparison with their lengths) from their undeformed state, which is assumed to be rectilinear. Then the stabilizer can be represented moderately well as a homogeneous elastic beam executing free vibrations about an equilibrium position, which may correspond to, either an undeformed state, or a slightly deformed one, depending on the character of the external fields acting on the satellite. The torsional vibrations of this beam can, of course, be neglected as a consequence of the large structural damping which occurs at the lap jointed edges of the tube.

Let us orient the axis $O_i y_{3i}$ of the system of axes $O_i y_{1i} y_{2i} y_{3i}$ along the bar S_i (Fig. 2). The displacement vector $\mathbf{u}_1$ of the point $y_{11}y_{21}y_{31}$ of bar S_1 resulting from the flexural deformation, can then be represented in the following way, on the basis of [5]:

$$\begin{aligned}
\mathbf{u}_1 = (\mathbf{i}_1\cos\gamma + \mathbf{i}_3\sin\gamma)\sum_{\alpha=1}^{n} q_\alpha\varphi_\alpha(y_{31}) + \mathbf{i}_2\sum_{\alpha=1}^{n} q_{n+\alpha}\psi_\alpha(y_{31}) + \\
+ (\mathbf{i}_1\sin\gamma - \mathbf{i}_3\cos\gamma)\Big[\sum_{\alpha=1}^{n} y_{11}q_\alpha\varphi_\alpha{}'(y_{31}) + \sum_{\alpha=1}^{n} y_{21}q_{n+\alpha}\psi_\alpha{}'(y_{31})\Big] + \\
+ \frac{1}{2}(\mathbf{i}_1\sin\gamma - \mathbf{i}_3\cos\gamma)\sum_{\alpha=1}^{n}\sum_{\beta=1}^{n}\Big[q_\alpha q_\beta\int_0^{y_{31}}\varphi_\alpha{}'(\xi)\varphi_\beta{}'(\xi)\,d\xi + \\
+ q_{n+\alpha}q_{n+\beta}\int_0^{y_{31}}\psi_\alpha{}'(\xi)\psi_\beta{}'(\xi)\,d\xi\Big]
\end{aligned} \tag{2.1}$$

where $\mathbf{i}_1$, $\mathbf{i}_2$, $\mathbf{i}_3$ are the unit vectors along the axes $Cx_1x_2x_3$ of the satellite, γ is the angle of mounting of the bars, $\varphi_\alpha(y_{31})$, and $\psi_\alpha(y_{31})$ are the normal modes of vibration of the bar S_1 in the planes $y_{11}y_{31}$ and $y_{21}y_{31}$, q_α and $q_{n+\alpha}$ are the systems of generalised coordinates which correspond to these modes. In Equation (2.1), both terms linear in generalised coordinates describing the deflection of the beam and the rotation of its cross-sections, and quadratic terms describing the distortion of the bent bar, are taken into account. Analogously, we have for S_2

$$\begin{aligned}
\mathbf{u}_2 = (-\mathbf{i}_1\cos\gamma + \mathbf{i}_3\sin\gamma)\sum_{\alpha=1}^{n} q_\alpha{}^*\varphi_\alpha{}^*(y_{32}) + \mathbf{i}_2\sum_{\alpha=1}^{n} q^*_{n+\alpha}\psi_\alpha{}^*(y_{32}) + \\
+ (\mathbf{i}_1\sin\gamma + \mathbf{i}_3\cos\gamma)\Big[\sum_{\alpha=1}^{n} y_{12}q_\alpha{}^*\varphi_\alpha{}^{*\prime}(y_{32}) + \sum_{\alpha=1}^{n} y_{22}q^*_{n+\alpha}\psi_\alpha{}^{*\prime}(y_{32})\Big] + \\
+ \frac{1}{2}(\mathbf{i}_1\sin\gamma + \mathbf{i}_3\cos\gamma)\sum_{\alpha=1}^{n}\sum_{\beta=1}^{n}\Big[q_\alpha{}^*q_\beta{}^*\int_0^{y_{32}}\varphi_\alpha{}^{*\prime}(\xi)\varphi_\beta{}^{*\prime}(\xi)\,d\xi + \\
+ q^*_{n+\alpha}q^*_{n+\beta}\int_0^{y_{32}}\psi_\alpha{}^{*\prime}(\xi)\psi_\beta{}^{*\prime}(\xi)\,d\xi\Big]
\end{aligned} \tag{2.2}$$

From Equations (2.1) and (2.2) the displacement vectors of the end masses are:

$$\begin{aligned}
\mathbf{u}_1(m_2) = (\mathbf{i}_1\cos\gamma + \mathbf{i}_3\sin\gamma)\sum_{\alpha=1}^{n} q_\alpha\varphi_\alpha(L) + \mathbf{i}_2\sum_{\alpha=1}^{n} q_{n+\alpha}\psi_\alpha(L) + \\
+ \frac{1}{2}(\mathbf{i}_1\sin\gamma - \mathbf{i}_3\cos\gamma)\sum_{\alpha=1}^{n}\sum_{\beta=1}^{n}\Big[q_\alpha q_\beta\int_0^{L}\varphi_\alpha{}'(\xi)\varphi_\beta{}'(\xi)\,d\xi + \\
+ q_{n+\alpha}q_{n+\beta}\int_0^{L}\psi_\alpha{}'(\xi)\psi_\beta{}'(\xi)\,d\xi\Big]
\end{aligned} \tag{2.3}$$

$$\begin{aligned}
\mathbf{u}_2(m_2) = (-\mathbf{i}_1\cos\gamma + \mathbf{i}_3\sin\gamma)\sum_{\alpha=1}^{n} q_\alpha{}^*\varphi_\alpha{}^*(L) + \mathbf{i}_2\sum_{\alpha=1}^{n} q^*_{n+\alpha}\psi_\alpha{}^*(L) + \\
+ \frac{1}{2}(\mathbf{i}_1\sin\gamma + \mathbf{i}_3\cos\gamma)\sum_{\alpha=1}^{n}\sum_{\beta=1}^{n}\Big[q_\alpha{}^*q_\beta{}^*\int_0^{L}\varphi_\alpha{}^{*\prime}(\xi)\varphi_\beta{}^{*\prime}(\xi)\,d\xi +
\end{aligned} \tag{2.4}$$

$$+ q_{n+\alpha}^{*}q_{n+\beta}^{*}\int_0^L \psi_\alpha^{*\prime}(\xi)\,\psi_\beta^{*\prime}(\xi)\,d\xi\,\Big]$$

In addition, the radius vectors of points of the bar S_i with coordinates y_{1i} y_{2i} y_{3i} are, in the undeformed state, determined by the expressions (Fig. 2)

$$\mathbf{r}_1 = \mathbf{i}_1(-R_2 + y_{11}\cos\gamma - y_{31}\sin\gamma) + \mathbf{i}_2 y_{21} + \mathbf{i}_3(R_1 + y_{11}\sin\gamma + y_{31}\cos\gamma) \tag{2.5}$$

$$\mathbf{r}_2 = \mathbf{i}_1(-R_2 - y_{12}\cos\gamma - y_{32}\sin\gamma) + \mathbf{i}_2 y_{22} + \mathbf{i}_3(-R_1 + y_{12}\sin\gamma - y_{32}\cos\gamma)$$

In what follows, by assuming no difference in the nature of the vibrations of the bars in the y_{1i} y_{3i} and y_{2i} y_{3i} planes, we may consider fo simplicity that

$$\varphi_\alpha(\xi) \equiv \psi_\alpha(\xi) \equiv \varphi_\alpha^*(\xi) \equiv \psi_\alpha^*(\xi) \tag{2.6}$$

Using the notations of (1.12), we have

for S_1

$$\mathbf{U}_1^{\alpha} = (\mathbf{i}_1\cos\gamma + \mathbf{i}_3\sin\gamma)\,\varphi_\alpha(y_{31}) + (\mathbf{i}_1\sin\gamma - \mathbf{i}_3\cos\gamma)\,y_{11}\varphi_\alpha'(y_{31}) \tag{2.7}$$

$$\mathbf{U}_1^{n+\alpha} = \mathbf{i}_2\varphi_\alpha(y_{31}) + (\mathbf{i}_1\sin\gamma - \mathbf{i}_3\cos\gamma)\,y_{21}\varphi_\alpha'(y_{31}), \qquad \mathbf{U}_1^{\alpha,\,n+\beta} = \mathbf{U}_1^{n+\alpha,\,\beta} = 0$$

$$\mathbf{U}_1^{\alpha\beta} = \mathbf{U}_1^{n+\alpha,\,n+\beta} = (\mathbf{i}_1\sin\gamma - \mathbf{i}_3\cos\gamma)\int_0^{y_{31}}\varphi_\alpha'(\xi)\,\varphi_\beta'(\xi)\,d\xi$$

for S_2

$$\mathbf{U}_2^{\alpha} = (-\mathbf{i}_1\cos\gamma + \mathbf{i}_3\sin\gamma)\,\varphi_\alpha(y_{32}) + (\mathbf{i}_1\sin\gamma + \mathbf{i}_3\cos\gamma)\,y_{12}\varphi_\alpha'(y_{32})$$

$$\mathbf{U}_2^{n+\alpha} = \mathbf{i}_2\varphi_\alpha(y_{32}) + (\mathbf{i}_1\sin\gamma + \mathbf{i}_3\cos\gamma)\,y_{22}\varphi_\alpha'(y_{32}), \quad \mathbf{U}_2^{\alpha,\,n+\beta} = \mathbf{U}_2^{n+\alpha,\beta} = 0 \tag{2.8}$$

$$\mathbf{U}_2^{\alpha\beta} = \mathbf{U}_2^{n+\alpha,\,n+\beta} = (\mathbf{i}_1\sin\gamma + \mathbf{i}_3\cos\gamma)\int_0^{y_{32}}\varphi_\alpha'(\xi)\,\varphi_\beta'(\xi)\,d\xi$$

In the calculations of (1.16) for the equations of motion (1.14) and (1.15), the integration may conveniently be carried out with respect to the dimensionless variable $s_i = y_{3i}/L$, where L is the length of a bar. As a result, we have

$$\mathbf{a}_1^{\alpha} = m^\alpha(\mathbf{i}_1\cos\gamma + \mathbf{i}_3\sin\gamma), \qquad \mathbf{a}_2^{\alpha} = m^\alpha(-\mathbf{i}_1\cos\gamma + \mathbf{i}_3\sin\gamma)$$

$$\mathbf{a}_1^{\alpha\beta} = \mathbf{a}_1^{n+\alpha,\,n+\beta} = m_*^{\alpha\beta}L^{-1}(\mathbf{i}_1\sin\gamma - \mathbf{i}_3\cos\gamma), \quad \mathbf{a}_i^{n+\alpha} = m^\alpha\,\mathbf{i}_2,$$

$$\mathbf{a}_2^{\alpha\beta} = \mathbf{a}_2^{n+\alpha,\,n+\beta} = m_*^{\alpha\beta}L^{-1}(\mathbf{i}_1\sin\gamma + \mathbf{i}_3\cos\gamma), \quad \mathbf{a}_i^{\alpha,\,n+\beta} = \mathbf{a}_i^{n+\alpha,\,\beta} = 0$$

$$m^\alpha = m_1 n^\alpha + m_2\varphi_\alpha(1), \qquad m_*^{\alpha\beta} = m_1 m^{\alpha\beta} + m_2 l^{\alpha\beta}$$

$$A^{\alpha\beta} = A_i^{\alpha\beta} = A_i^{n+\alpha,\,n+\beta} = m_1 A_*^{\alpha\beta} + m_2\varphi_\alpha(1)\,\varphi_\beta(1), \qquad A_i^{\alpha,\,n+\beta} = A_i^{n+\alpha,\,\beta} = 0$$

$$\mathbf{G}_i^{\alpha} = g^\alpha\mathbf{i}_2, \quad \mathbf{G}_1^{n+\alpha} = -\mathbf{i}_1 g_1^{n+\alpha} - \mathbf{i}_3 g_2^{n+\alpha}, \qquad \mathbf{G}_2^{n+\alpha} = \mathbf{i}_1 g_1^{n+\alpha} - \mathbf{i}_3 g_2^{n+\alpha}$$

$$g_1^{n+\alpha} = g_1^{\alpha} = m_1(R_1 n^\alpha + L\cos\gamma\, n_*^{\alpha}) + m_2(R_1 + L\cos\gamma)\,\varphi_\alpha(1)$$

$$g_2^{n+\alpha} = g_2^{\alpha} = m_1(R_2 n^\alpha + L\sin\gamma\, n_*^{\alpha}) + m_2(R_2 + L\sin\gamma)\,\varphi_\alpha(1) \tag{2.9}$$

$$g^\alpha = \cos\gamma\, g_1{}^\alpha + \sin\gamma g_2{}^\alpha, \qquad \Gamma_i{}^{\alpha\beta} = \Gamma_i{}^{n+\alpha,\, n+\beta} = 0$$

$$\Gamma_1{}^{\alpha,\, n+\beta} = A^{\alpha\beta}(\mathbf{i}_3\cos\gamma - \mathbf{i}_1\sin\gamma), \qquad \Gamma_2{}^{\alpha,\, n+\beta} = -A^{\alpha\beta}(\mathbf{i}_3\cos\gamma + \mathbf{i}_1\sin\gamma)$$

$$\mathbf{G}_1{}^{\beta\alpha} = \mathbf{G}_1{}^{n+\beta,\, n+\alpha} = G^{\alpha\beta}\mathbf{i}_2, \qquad \mathbf{G}_2{}^{\beta\alpha} = \mathbf{G}_2{}^{n+\beta,\, n+\alpha} = -G^{\alpha\beta}\mathbf{i}_2$$

$$G^{\alpha\beta} = m_*{}^{\alpha\beta}L^{-1}(R_1\sin\gamma - R_2\cos\gamma)$$

$$\mathbf{G}_1{}^{n+\beta,\, \alpha} = \Gamma_1{}^{n+\beta,\, \alpha} = -A^{\alpha\beta}(\mathbf{i}_3\cos\gamma - \mathbf{i}_1\sin\gamma)$$

$$\mathbf{G}_2{}^{n+\beta,\, \alpha} = \Gamma_2{}^{n+\beta,\, \alpha} = A^{\alpha\beta}(\mathbf{i}_3\cos\gamma + \mathbf{i}_1\sin\gamma)$$

$$\Lambda_1{}^\alpha = \mathbf{i}_1\mathbf{i}_1\Lambda_{11}{}^\alpha + \mathbf{i}_2\mathbf{i}_2\Lambda_{22}{}^\alpha + \mathbf{i}_3\mathbf{i}_3\Lambda_{33}{}^\alpha - (\mathbf{i}_1\mathbf{i}_3 + \mathbf{i}_3\mathbf{i}_1)\Lambda_{13}{}^\alpha \tag{2.9}$$

$$\Lambda_1{}^{n+\alpha} = -(\mathbf{i}_1\mathbf{i}_2 + \mathbf{i}_2\mathbf{i}_1)\Lambda_{12}{}^{n+\alpha} - (\mathbf{i}_3\mathbf{i}_2 + \mathbf{i}_2\mathbf{i}_3)\Lambda_{23}{}^{n+\alpha}$$

$$\Lambda_2{}^\alpha = -\mathbf{i}_1\mathbf{i}_1\Lambda_{11}{}^\alpha - \mathbf{i}_2\mathbf{i}_2\Lambda_{22}{}^\alpha - \mathbf{i}_3\mathbf{i}_1\Lambda_{33}{}^\alpha - (\mathbf{i}_1\mathbf{i}_3 + \mathbf{i}_3\mathbf{i}_1)\Lambda_{13}{}^\alpha$$

$$\Lambda_2{}^{n+\alpha} = -(\mathbf{i}_1\mathbf{i}_2 + \mathbf{i}_2\mathbf{i}_1)\Lambda_{12}{}^{n+\alpha} + (\mathbf{i}_3\mathbf{i}_2 + \mathbf{i}_2\mathbf{i}_3)\Lambda_{23}{}^{n+\alpha}$$

$$\Lambda_{11}{}^\alpha = \sin\gamma g_1{}^\alpha, \qquad \Lambda_{33}{}^\alpha = -\cos\gamma g_2{}^\alpha, \qquad \Lambda_{13}{}^\alpha = \tfrac{1}{2}(\cos\gamma g_1{}^\alpha - \sin\gamma g_2{}^\alpha)$$

$$\Lambda_{22}{}^\alpha = m^\alpha(R_1\sin\gamma - R_2\cos\gamma), \qquad \Lambda_{12}{}^{n+\alpha} = -\tfrac{1}{2}g_2{}^\alpha, \qquad \Lambda_{23}{}^{n+\alpha} = \tfrac{1}{2}g_1{}^\alpha$$

$$Q_1{}^{\alpha,\, n+\beta} = Q_1{}^{n+\alpha,\, \beta} = -\tfrac{1}{2}A^{\alpha\beta}[(\mathbf{i}_1\mathbf{i}_2 + \mathbf{i}_2\mathbf{i}_1)\cos\gamma + (\mathbf{i}_2\mathbf{i}_3 + \mathbf{i}_3\mathbf{i}_2)\sin\gamma]$$

$$Q_2{}^{\alpha,\, n+\beta} = Q_2{}^{n+\alpha,\, \beta} = \tfrac{1}{2}A^{\alpha\beta}[(\mathbf{i}_1\mathbf{i}_2 + \mathbf{i}_2\mathbf{i}_1)\cos\gamma - (\mathbf{i}_2\mathbf{i}_3 + \mathbf{i}_3\mathbf{i}_2)\sin\gamma]$$

$$Q_1{}^{\alpha\beta} = \mathbf{i}_1\mathbf{i}_1Q_{11}{}^{\alpha\beta} + \mathbf{i}_2\mathbf{i}_2Q_{22}{}^{\alpha\beta} + \mathbf{i}_3\mathbf{i}_3Q_{33}{}^{\alpha\beta} - (\mathbf{i}_1\mathbf{i}_3 + \mathbf{i}_3\mathbf{i}_1)Q_{13}{}^{\alpha\beta}$$

$$Q_2{}^{\alpha\beta} = \mathbf{i}_1\mathbf{i}_1Q_{11}{}^{\alpha\beta} + \mathbf{i}_2\mathbf{i}_2Q_{22}{}^{\alpha\beta} + \mathbf{i}_3\mathbf{i}_3Q_{33}{}^{\alpha\beta} + (\mathbf{i}_1\mathbf{i}_3 + \mathbf{i}_3\mathbf{i}_1)Q_{13}{}^{\alpha\beta}$$

$$Q_1{}^{n+\alpha,\, n+\beta} = \mathbf{i}_1\mathbf{i}_1Q_{11}{}^{n+\alpha,\, n+\beta} + \mathbf{i}_2\mathbf{i}_2Q_{22}{}^{n+\alpha,\, n+\beta} + \mathbf{i}_3\mathbf{i}_3Q_{33}{}^{n+\alpha,\, n+\beta} - \\ - (\mathbf{i}_1\mathbf{i}_3 + \mathbf{i}_3\mathbf{i}_1)Q_{13}{}^{n+\alpha,\, n+\beta}$$

$$Q_2{}^{n+\alpha,\, n+\beta} = \mathbf{i}_1\mathbf{i}_1Q_{11}{}^{n+\alpha,\, n+\beta} + \mathbf{i}_2\mathbf{i}_2Q_{22}{}^{n+\alpha,\, n+\beta} + \mathbf{i}_3\mathbf{i}_3Q_{33}{}^{n+\alpha,\, n+\beta} + \\ + (\mathbf{i}_1\mathbf{i}_3 + \mathbf{i}_3\mathbf{i}_1)Q_{13}{}^{n+\alpha,\, n+\beta}$$

$$Q_{11}{}^{\alpha\beta} = \sin^2\gamma A^{\alpha\beta} - \cos\gamma\, d_1{}^{\alpha\beta}, \quad Q_{33}{}^{\alpha\beta} = \cos^2\gamma A^{\alpha\beta} - \sin\gamma\, d_2{}^{\alpha\beta}$$

$$Q_{22}{}^{\alpha\beta} = A^{\alpha\beta} - \sin\gamma\, d_2{}^{\alpha\beta} - \cos\gamma\, d_1{}^{\alpha\beta}$$

$$Q_{13}{}^{\alpha\beta} = \sin\gamma\cos\gamma A^{\alpha\beta} + \tfrac{1}{2}[\cos\gamma d_2{}^{\alpha\beta} + \sin\gamma\, d_1{}^{\alpha\beta}]$$

$$Q_{11}{}^{n+\alpha,\, n+\beta} = A^{\alpha\beta} - \cos\gamma\, d_1{}^{\alpha\beta}, \quad Q_{33}{}^{n+\alpha,\, n+\beta} = A^{\alpha\beta} - \sin\gamma\, d_2{}^{\alpha\beta}$$

$$Q_{22}{}^{n+\alpha,\, n+\beta} = -\sin\gamma\, d_2{}^{\alpha\beta} - \cos\gamma\, d_1{}^{\alpha\beta}$$

$$Q_{13}{}^{n+\alpha,\, n+\beta} = \tfrac{1}{2}[\cos\gamma\, d_2{}^{\alpha\beta} + \sin\gamma\, d_1{}^{\alpha\beta}]$$

$$d_1{}^{\alpha\beta} = L^{-1}[m_1(R_1m^{\alpha\beta} + L\cos\gamma m_{**}^{\alpha\beta}) + m_2(R_1 + L\cos\gamma)l^{\alpha\beta}]$$

$$d_2{}^{\alpha\beta} = L^{-1}[m_1(R_2m^{\alpha\beta} + L\sin\gamma m_*^{\alpha\beta}) + m_2(R_2 + L\sin\gamma)l^{\alpha\beta}]$$

where the following notations are used for integrals:

$$n^\alpha = \int_0^1 \varphi_\alpha(s)\,ds, \qquad n_*{}^\alpha = \int_0^1 s\varphi_\alpha(s)\,ds, \qquad l^\alpha = \int_0^1 \varphi_\alpha{}'(s)\,ds \tag{2.10}$$

$$A_*^{\alpha\beta} = \int_0^1 \varphi_\alpha(s)\varphi_\beta(s)\,ds, \qquad l^{\alpha\beta} = \int_0^1 \varphi_\alpha'(s)\varphi_\beta'(s)\,ds$$

$$m^{\alpha\beta} = \int_0^1 ds \int_0^S \varphi_\alpha'(\sigma)\varphi_\beta'(\sigma)\,d\sigma, \qquad m_*^{\alpha\beta} = \int_0^1 s\,ds \int_0^S \varphi_\alpha'(\sigma)\varphi_\beta'(\sigma)\,d\sigma$$

Moreover, in the computation of (2.9) the terms, in which the rotations of the cross sections of the stabilizer were included were neglected.

From Equations (1.14) and (2.9), the equations for rotation of the satellite about its center of inertia C referred to the $x_1x_2x_3$ axes, have the following form:

$$\begin{aligned}
&A\omega_1^{\cdot} + (C-B)\omega_2\omega_3 + 2\omega_1^{\cdot}\sum_{\alpha=1}^{n}\Lambda_{11}^{\alpha}y_\alpha - 2(\omega_2^{\cdot} - \omega_1\omega_3)\sum_{\alpha=1}^{n}\Lambda_{12}^{n+\alpha}x_{n+\alpha} - \\
&- 2(\omega_3^{\cdot} + \omega_1\omega_2)\sum_{\alpha=1}^{n}\Lambda_{13}^{\alpha}x_\alpha + 2\omega_2\omega_3\sum_{\alpha=1}^{n}(\Lambda_{33}^{\alpha} - \Lambda_{22}^{\alpha})y_\alpha + \\
&+ 2(\omega_3^2 - \omega_2^2)\sum_{\alpha=1}^{n}\Lambda_{23}^{\alpha}y_{n+\alpha} - \sum_{\alpha=1}^{n}g_1^{\alpha}\ddot{y}_{n+\alpha} + 2\omega_1\sum_{\alpha=1}^{n}\Lambda_{11}^{\alpha}y_\alpha^{\cdot} - \\
&- \omega_3\sum_{\alpha=1}^{n}(2\Lambda_{13}^{\alpha} + g^{\alpha})x_\alpha^{\cdot} + (\dot{V}_{C3} + \omega_1V_{C2} - \omega_2V_{C1})\sum_{\alpha=1}^{n}m^{\alpha}x_{n+\alpha} - \\
&- \sin\gamma(\dot{V}_{C2} + \omega_3V_{C1} - \omega_1V_{C3})\sum_{\alpha=1}^{n}m^{\alpha}x_\alpha = m_1^C + M_1^C
\end{aligned}\tag{2.11}$$

$$\begin{aligned}
&B\omega_2^{\cdot} + (A-C)\omega_1\omega_3 + 2\omega_2^{\cdot}\sum_{\alpha=1}^{n}\Lambda_{22}^{\alpha}y_\alpha - 2(\omega_1^{\cdot} + \omega_2\omega_3)\sum_{\alpha=1}^{n}\Lambda_{12}^{n+\alpha}x_{n+\alpha} - \\
&- 2(\omega_3^{\cdot} - \omega_1\omega_2)\sum_{\alpha=1}^{n}\Lambda_{23}^{n+\alpha}y_{n+\alpha} + 2\omega_1\omega_3\sum_{\alpha=1}^{n}(\Lambda_{11}^{\alpha} - \Lambda_{33}^{\alpha})y_\alpha - \\
&- 2(\omega_1^2 - \omega_3^2)\sum_{\alpha=1}^{n}\Lambda_{13}^{\alpha}x_\alpha + \sum_{\alpha=1}^{n}g^{\alpha}x_\alpha^{\cdot\cdot} + 2\omega_2\sum_{\alpha=1}^{n}\Lambda_{22}^{\alpha}y_\alpha^{\cdot} - \\
&- \omega_1\sum_{\alpha=1}^{n}(2\Lambda_{12}^{n+\alpha} - g_2^{\alpha})\dot{x}_{n+\alpha} - \omega_3\sum_{\alpha=1}^{n}(2\Lambda_{23}^{n+\alpha} + g_1^{\alpha})\dot{y}_{n+\alpha} + \\
&+ \sin\gamma(\dot{V}_{C1} + \omega_2V_{C3} - \omega_3V_{C2})\sum_{\alpha=1}^{n}m^{\alpha}x_\alpha - \cos\gamma(\dot{V}_{C3} + \omega_1V_{C2} - \omega_2V_{C1})\times \\
&\times\sum_{\alpha=1}^{n}m^{\alpha}y_\alpha = m_2^C + M_2^C
\end{aligned}\tag{2.12}$$

$$\begin{aligned}
&C\omega_3^{\cdot} + (B-A)\omega_1\omega_2 + 2\omega_3^{\cdot}\sum_{\alpha=1}^{n}\Lambda_{33}^{\alpha}y_\alpha - 2(\omega_1^{\cdot} - \omega_2\omega_3)\sum_{\alpha=1}^{n}\Lambda_{13}^{\alpha}x_\alpha - \\
&- 2(\omega_2^{\cdot} + \omega_1\omega_3)\sum_{\alpha=1}^{n}\Lambda_{23}^{n+\alpha}y_{n+\alpha} + 2\omega_1\omega_2\sum_{\alpha=1}^{n}(\Lambda_{22}^{\alpha} - \Lambda_{11}^{\alpha})y_\alpha + \\
&+ 2(\omega_2^2 - \omega_1^2)\sum_{\alpha=1}^{n}\Lambda_{12}^{n+\alpha}x_{n+\alpha} - \sum_{\alpha=1}^{n}g_2^{\alpha}\ddot{x}_{n+\alpha} + 2\omega_3\sum_{\alpha=1}^{n}\Lambda_{33}^{\alpha}y_\alpha^{\cdot} -
\end{aligned}\tag{2.13}$$

$$-\omega_3 \sum_{\alpha=1}^{n} (2\Lambda_{11}{}^{\alpha} - g^{\alpha}) x_{\alpha}{}^{\cdot} - (V_{C1}^{*} + \omega_2 V_{C3} - \omega_3 V_{C2}) \sum_{\alpha=1}^{n} m^{\alpha} x_{n+\alpha} +$$

$$+ \cos\gamma (V_{C2}^{*} + \omega_3 V_{C1} - \omega_1 V_{C3}) \sum_{\alpha=1}^{n} m^{\alpha} y_{\alpha} = m_3{}^C + M_3{}^C$$

In Equations (2.11) to (2.13), the following notation was introduced for new generalised coordinates:

$$x_{\alpha} = q_{\alpha} + q_{\alpha}{}^{*}, \quad x_{n+\alpha} = q_{n+\alpha} + q_{n+\alpha}^{*}, \quad y_{\alpha} = q_{\alpha} - q_{\alpha}{}^{*}, \quad y_{n+\alpha} = q_{n+\alpha} - q_{n+\alpha}^{*} \tag{2.14}$$

In scalar notation, Equation (1.15) has now the form

$$\sum_{\beta=1}^{n} A^{\alpha\beta} x_{\beta}{}^{\cdot\cdot} = Q_{\alpha} + Q_{\alpha}{}^{*} - 2\sin\gamma (V_{C3}^{*} + \omega_1 V_{C2} - \omega_2 V_{C1}) m^{\alpha} -$$

$$- \sin\gamma (V_{C1}^{*} + \omega_2 V_{C3} - \omega_3 V_{C2}) \frac{1}{L} \sum_{\beta=1}^{n} m_{*}{}^{\alpha\beta} x_{\beta} -$$

$$- \cos\gamma (V_{C3}^{*} + \omega_1 V_{C2} - \omega_2 V_{C1}) \frac{1}{L} \sum_{\beta=1}^{n} m_{*}{}^{\alpha\beta} y_{\beta} - 4\omega_1\omega_3\Lambda_{13}{}^{\alpha} - 2g^{\alpha}\omega_2{}^{\cdot} +$$

$$+ \sum_{\beta=1}^{n} [(\omega_1{}^2 Q_{11}{}^{\alpha\beta} + \omega_2{}^2 Q_{22}{}^{\alpha\beta} + \omega_3{}^2 Q_{33}{}^{\alpha\beta}) x_{\beta} - 2\omega_1\omega_3 Q_{13}{}^{\alpha\beta} y_{\beta}] -$$

$$- \sin\gamma (\omega_1{}^{\cdot} + \omega_2\omega_3) \sum_{\beta=1}^{n} A^{\alpha\beta} x_{n+\beta} + \cos\gamma (\omega_3{}^{\cdot} - \omega_1\omega_2) \sum_{\beta=1}^{n} A^{\alpha\beta} y_{n+\beta} +$$

$$+ 2 \sum_{\beta=1}^{n} A^{\alpha\beta} [\cos\gamma\, \omega_3 y_{n+\beta}^{\cdot} - \sin\gamma\, \omega_1 x_{n+\beta}^{\cdot}] - \omega_2{}^{\cdot} \sum_{\beta=1}^{n} G^{\alpha\beta} y_{\beta} \tag{2.15}$$

$$\sum_{\beta=1}^{n} A^{\alpha\beta} y_{\beta}{}^{\cdot\cdot} = Q_{\alpha} - Q_{\alpha}{}^{*} - 2\cos\gamma (V_{C1}^{*} + \omega_2 V_{C3} - \omega_3 V_{C2}) m^{\alpha} -$$

$$- \sin\gamma (V_{C1}^{*} + \omega_2 V_{C3} - \omega_3 V_{C2}) \frac{1}{L} \sum_{\beta=1}^{n} m_{*}{}^{\alpha\beta} y_{\beta} - \cos\gamma (V_{C3}^{*} + \omega_1 V_{C2} -$$

$$- \omega_2 V_{C1}) \frac{1}{L} \sum_{\beta=1}^{n} m_{*}{}^{\alpha\beta} x_{\beta} + 2(\omega_1{}^2 \Lambda_{11}{}^{\alpha} + \omega_2{}^2 \Lambda_{22}{}^{\alpha} + \omega_3{}^2 \Lambda_{33}{}^{\alpha}) + 2g_2{}^{\alpha}\omega_3{}^{\cdot} +$$

$$+ \sum_{\beta=1}^{n} [(\omega_1{}^2 Q_{11}{}^{\alpha\beta} + \omega_2{}^2 Q_{22}{}^{\alpha\beta} + \omega_3{}^2 Q_{33}{}^{\alpha\beta}) y_{\beta} - 2\omega_1\omega_3 Q_{13}{}^{\alpha\beta} x_{\beta}] +$$

$$+ \cos\gamma (\omega_3{}^{\cdot} - \omega_1\omega_2) \sum_{\beta=1}^{n} A^{\alpha\beta} x_{n+\beta} - \sin\gamma (\omega_1{}^{\cdot} + \omega_2\omega_3) \sum_{\beta=1}^{n} A^{\alpha\beta} y_{n+\beta} +$$

$$+ 2 \sum_{\beta=1}^{n} A^{\alpha\beta} [\cos\gamma\, \omega_3 x_{n+\beta}^{\cdot} - \sin\gamma\, \omega_1 y_{n+\beta}^{\cdot}] - \omega_2{}^{\cdot} \sum_{\beta=1}^{n} G^{\alpha\beta} x_{\beta}$$

$$\sum_{\beta=1}^{n} A^{\alpha\beta} x_{n+\beta}^{\cdot\cdot} = Q_{n+\alpha} + Q_{n+\alpha}^{*} - 2m^{\alpha} (V_{C2}^{*} + \omega_3 V_{C1} - \omega_1 V_{C3}) -$$

$$- \sin\gamma (V_{C1}^{*} + \omega_2 V_{C3} - \omega_3 V_{C2}) \frac{1}{L} \sum_{\beta=1}^{n} m_{*}{}^{\alpha\beta} x_{n+\beta} +$$

$$
\begin{aligned}
&+\cos\gamma\,(\dot V_{C3}+\omega_1 V_{C2}-\omega_2 V_{C1})\frac{1}{L}\sum_{\beta=1}^{n} m_*{}^{\alpha\beta}y_{n+\beta}+2g_2{}^{\alpha}(\omega_3{}^{\cdot}+\omega_1\omega_2)+ \\
&+\sum_{\beta=1}^{n}[(\omega_1{}^2Q_{11}{}^{n+\alpha,\,n+\beta}+\omega_2{}^2Q_{22}{}^{n+\alpha,\,n+\beta}+\omega_3{}^2Q_{33}{}^{n+\alpha,\,n+\beta})\,x_{n+\beta}- \\
&-2\omega_1\omega_3Q_{13}{}^{n+\alpha,\,n+\beta}y_{n+\beta}]-\sin\gamma\,(\omega_1{}^{\cdot}+\omega_2\omega_3)\sum_{\beta=1}^{n}A^{\alpha\beta}x_\beta+\cos\gamma\,(\omega_3{}^{\cdot}- \\
&-\omega_1\omega_2)\sum_{\beta=1}^{n}A^{\alpha\beta}y_\beta+2\sum_{\beta=1}^{n}A^{\alpha\beta}\,[\sin\gamma\,\omega_1 x_\beta{}^{\cdot}-\cos\gamma\,\omega_3 y_\beta{}^{\cdot}]-\omega_2{}^{\cdot}\sum_{\beta=1}^{n}G^{\alpha\beta}y_{n+\beta}
\end{aligned}
\tag{2.15}
$$

$$
\begin{aligned}
\sum_{\beta=1}^{n}A^{\alpha\beta}\ddot y_{n+\beta}&=Q_{n+\alpha}-Q_{n+\alpha}^{\bullet}+\cos\gamma\,(\dot V_{C3}+\omega_1V_{C2}-\omega_2V_{C1})\frac{1}{L}\sum_{\beta=1}^{n}m_*{}^{\alpha\beta}x_{n+\beta}- \\
&-\sin\gamma\,(\dot V_{C1}+\omega_2V_{C3}-\omega_3V_{C2})\frac{1}{L}\sum_{\rho=1}^{n}m_*{}^{\alpha\beta}y_{n+\beta}+2g_1{}^{\alpha}(\omega_1{}^{\cdot}-\omega_2\omega_3)+ \\
&+\sum_{\beta=1}^{n}[(\omega_1{}^2Q_{11}{}^{n+\alpha,\,n+\beta}+\omega_2{}^2Q_{22}{}^{n+\alpha,\,n+\beta}+\omega_3{}^2Q_{33}{}^{n+\alpha,\,n+\beta})\,y_{n+\beta}- \\
&-2\omega_1\omega_3Q_{13}{}^{n+\alpha,\,n+\beta}x_{n+\beta}]+\cos\gamma\,(\omega_3{}^{\cdot}-\omega_1\omega_2)\sum_{\beta=1}^{n}A^{\alpha\beta}x_\beta-\sin\gamma\,(\omega_1{}^{\cdot}+ \\
&+\omega_2\omega_3)\sum_{\beta=1}^{n}A^{\alpha\beta}y_\beta+2\sum_{\beta=1}^{n}A^{\alpha\beta}\,[\sin\gamma\,\omega_1y_\beta{}^{\cdot}-\cos\gamma\,\omega_3x_\beta{}^{\cdot}]-\omega_2{}^{\cdot}\sum_{\beta=1}^{n}G^{\alpha\beta}x_{n+\beta} \\
&(\alpha=1,2,\ldots,n)
\end{aligned}
$$

Thus we have obtained a system of equations, which describes the motion of the deformable satellite as a system with a finite number $4n+3$, of degrees of freedom ($N=4n$). In Equations (2.11) to (2.13) the component of the vector $\mathbf{m}^C$ representing the principal moment of the gravitational forces acting in the central field of spherical Earth on the deformable system of the satellite, is determined by the approximate relation [6]

$$
\mathbf{m}_g{}^C\approx-\frac{\mu}{r_{OC}^2}\int(\mathbf{r}+\mathbf{u})\times\mathbf{k}^*\,dm-3\frac{\mu}{r_{OC}^3}\mathbf{k}^*\cdot(\boldsymbol{\Theta}^C+\boldsymbol{\Theta}^{C+})\times\mathbf{k}^* \tag{2.16}
$$

where r_{OC} is the distance between the center of attraction O and the point C, μ is the gravitational constant, and $\mathbf{k}^*$ is the unit vector in the direction of r_{OC}, defined by

$$
\mathbf{k}^*=\delta_1\mathbf{i}_1+\delta_2\mathbf{i}_2+\delta_3\mathbf{i}_3 \tag{2.17}
$$

Taking the expression (1.12) for $\mathbf{u}$ into account, we have, according to (1.16)

$$
\begin{aligned}
\mathbf{m}_g{}^C\approx{}&\frac{\mu}{r_{OC}^2}\mathbf{k}^*\times\int\mathbf{r}\,dm-3\frac{\mu}{r_{OC}^3}\mathbf{k}^*\cdot\boldsymbol{\Theta}^C\times\mathbf{k}^*+\frac{\mu}{r_{OC}^3}\mathbf{k}^*\times\sum_{\alpha=1}^{N}\mathbf{a}^{\alpha}q_\alpha- \\
&-6\frac{\mu}{r_{OC}^3}\mathbf{k}^*\cdot\sum_{\alpha=1}^{N}\Lambda^{\alpha}q_\alpha\times\mathbf{k}^*
\end{aligned}
\tag{2.18}
$$

For the problem under consideration, we have from (2.18) the following expression for the projections of $m^C{}_g$ on the axes $x_1x_2x_3$:

$$m_1^{(g)} \approx 3\frac{\mu}{r_{\bar{O}C}^3}(C-B)\,\delta_2\delta_3 + \frac{\mu}{r_{\bar{O}C}^2}\sum_{\alpha=1}^{n} m^\alpha\left[\delta_2 \sin\gamma x_\alpha - \delta_3 x_{n+\alpha}\right] -$$
$$- 6\frac{\mu}{r_{\bar{O}C}^3}\sum_{\alpha=1}^{n}\Big[\delta_2\delta_3\left(\Lambda_{22}{}^\alpha - \Lambda_{33}{}^\alpha\right) y_\alpha + \delta_1\delta_2\Lambda_{13}{}^\alpha x_\alpha +$$
$$+ \left(\delta_2{}^2 - \delta_3{}^2\right)\Lambda_{23}{}^{n+\alpha} y_{n+\alpha} - \delta_1\delta_3\Lambda_{12}{}^{n+\alpha} x_{n+\alpha}\Big]$$

$$m_2^{(g)} \approx 3\frac{\mu}{r_{\bar{O}C}^3}(A-C)\,\delta_1\delta_3 + \frac{\mu}{r_{\bar{O}C}^2}\sum_{\alpha=1}^{n} m^\alpha\left[-\delta_1 \sin\gamma x_\alpha + \delta_3\cos\gamma y_\alpha\right] -$$
$$- 6\frac{\mu}{r_{\bar{O}C}^3}\sum_{\alpha=1}^{n}\Big[\delta_1\delta_3\left(\Lambda_{33}{}^\alpha - \Lambda_{11}{}^\alpha\right) y_\alpha + \delta_2\delta_3\Lambda_{12}{}^{n+\alpha} x_{n+\alpha} + \qquad (2.19)$$
$$+ \left(\delta_3{}^2 - \delta_1{}^2\right)\Lambda_{13}{}^{\alpha} x_{\alpha} - \delta_1\delta_2\Lambda_{23}{}^{n+\alpha} y_{n+\alpha}\Big]$$

$$m_3^{(g)} \approx 3\frac{\mu}{r_{\bar{O}C}^3}(B-A)\,\delta_1\delta_2 + \frac{\mu}{r_{\bar{O}C}^2}\sum_{\alpha=1}^{n} m^\alpha\left[-\delta_2\cos\gamma y_\alpha + \delta_1 x_{n+\alpha}\right] -$$
$$- 6\frac{\mu}{r_{OC}^3}\sum_{\alpha=1}^{n}\Big[\delta_1\delta_2\left(\Lambda_{11}{}^\alpha - \Lambda_{22}{}^\alpha\right) y_\alpha + \delta_1\delta_3\Lambda_{23}{}^{n+\alpha} y_{n+\alpha} +$$
$$+ \left(\delta_1{}^2 - \delta_2{}^2\right)\Lambda_{12}{}^{n+\alpha} x_{n+\alpha} - \delta_2\delta_3\Lambda_{13}{}^{\alpha} x_{\alpha}\Big]$$

The generalized forces in Equations (2.15) can be represented as

$$Q_\alpha = Q_\alpha^+ + F_\alpha + \Phi_\alpha \qquad (\alpha = 1,\dots,\ N = 4n) \qquad (2.20)$$

where Q_α^+, F_α and F_α, and Φ_α are the generalized forces due to the external forces, the elastic reactions, and the inelastic reactions of the bars.

The generalized forces due to the external forces (particularly to the gravitational forces) can be obtained as the coefficients of variations of the corresponding coordinates in the expression for the virtual work done by the external forces which resulted in the virtual displacement $\delta\mathbf{u}$ of the points of the body only as the result of deformations of the structure. The virtual work of the gravitational forces over the displacement $\delta\mathbf{u}$ is determined by the approximate expression

$$\delta A^{(g)} \approx -\frac{\mu}{r_{\bar{O}C}^2}\int\left[1 - 3\frac{\mathbf{r}+\mathbf{u}}{r_{OC}}\cdot\mathbf{k}^*\right]\mathbf{k}^*\cdot\delta\mathbf{u}\,dm - \frac{\mu}{r_{OC}^3}\int(\mathbf{r}+\mathbf{u})\cdot\delta\mathbf{u}\,dm \qquad (2.21)$$

Since, by (1.12)

$$\delta\mathbf{u} = \sum_{\alpha=1}^{N}\mathbf{U}^\alpha\delta q_\alpha + \sum_{\alpha=1}^{n}\sum_{\beta=1}^{n}\mathbf{U}^{\alpha\beta} q_\beta\delta q_\alpha$$

we obtain from Equation (2.21), the following expressions for the $Q_\alpha^{(g)}$ ($\alpha = 1, 2, \dots, N$), taking account only of the terms which are linear in the generalized coordinates q_α:

$$Q_\alpha{}^{(g)} \approx -\frac{\mu}{r_{\bar{O}C}^2}\mathbf{k}^*\cdot\left[\mathbf{a}^\alpha + \sum_{\beta=1}^{N}\mathbf{a}^{\alpha\beta}q_\beta\right] + \qquad (2.22)$$

$$+ 3\frac{\mu}{r^3_{\bar{O}C}}\int\Big\{(\mathbf{r}\cdot\mathbf{k}^*)\Big[\mathbf{U}^\alpha\cdot\mathbf{k}^* + \sum_{\beta=1}^{N}(\mathbf{U}^{\alpha\beta}\cdot\mathbf{k}^*)\,q_\beta\Big] +$$

$$+\sum_{\beta=1}^{N} q_\beta\,(\mathbf{U}^\alpha\cdot\mathbf{k}^*)\,(\mathbf{U}^\beta\cdot\mathbf{k}^*)\Big\}\,dm - \frac{\mu}{r^3_{\bar{O}C}}\int\Big[\mathbf{r}\cdot\Big(\mathbf{U}^\alpha + \sum_{\beta=1}^{n}\mathbf{U}^{\alpha\beta}q_\beta\Big) + \sum_{\beta=1}^{N}A^{\alpha\beta}q_\beta\Big]\,dm$$

On the basis of (2.22), we have in (2.15)

$$Q_\alpha^{(g)} + Q_\alpha^{*(g)} \approx -\frac{\mu}{r^2_{\bar{O}C}}\Big[2m^\alpha\sin\gamma\delta_3 + \frac{1}{L}\sum_{\beta=1}^{n} m_*^{\alpha\beta}(\sin\gamma\delta_1 x_\beta - \cos\gamma\delta_3 y_\beta)\Big] +$$

$$+ 3\frac{\mu}{r^3_{\bar{O}C}}\sum_{\beta=1}^{n}\Big\{-(d_2^{\alpha\beta}\delta_1^2\sin\gamma + d_1^{\alpha\beta}\delta_3^2\cos\gamma)\,x_\beta + \delta_1\delta_3\,(d_2^{\alpha\beta}\cos\gamma +$$

$$+ d_1^{\alpha\beta}\sin\gamma)\,y_\beta + A^{\alpha\beta}\Big[(\delta_1^2\cos^2\gamma + \delta_3^2\sin^2\gamma)\,x_\beta + 2\delta_1\delta_3\cos\gamma\sin\gamma y_\beta +$$

$$+ \delta_2\,(\delta_1\cos\gamma y_{n+\beta} + \delta_3\sin\gamma x_{n+\beta})\Big]\Big\} + \frac{\mu}{r^3_{\bar{O}C}}\Big[-\sum_{\beta=1}^{n} Q_{22}^{\alpha\beta}x_\beta +$$

$$+ 6\delta_1\delta_3\,(\cos\gamma g_1^\alpha - \sin\gamma g_2^\alpha)\Big] \tag{2.23}$$

$$Q_\alpha^{(g)} - Q_\alpha^{*(g)} \approx -\frac{\mu}{r^2_{\bar{O}C}}\Big[2m^\alpha\cos\gamma\delta_1 + \frac{1}{L}\sum_{\beta=1}^{n} m_*^{\alpha\beta}(\sin\gamma\delta_1 y_\beta - \cos\gamma\delta_3 x_\beta)\Big] +$$

$$+ 3\frac{\mu}{r^3_{\bar{O}C}}\sum_{\beta=1}^{n}\Big\{-(d_2^{\alpha\beta}\delta_1^2\sin\gamma + d_1^{\alpha\beta}\delta_3^2\cos\gamma)\,y_\beta + \delta_1\delta_3\,(d_2^{\alpha\beta}\cos\gamma +$$

$$+ d_1^{\alpha\beta}\sin\gamma)\,x_\beta + A^{\alpha\beta}\Big[(\delta_1^2\cos^2\gamma + \delta_3^2\sin^2\gamma)\,y_\beta + 2\delta_1\delta_3\cos\gamma\sin\gamma x_\beta +$$

$$+ \delta_2\,(\delta_1\cos\gamma x_{n+\beta} + \delta_3\sin\gamma y_{n+\beta})\Big]\Big\} + \frac{\mu}{r^3_{\bar{O}C}}\Big[-\sum_{\beta=1}^{n} Q_{22}^{\alpha\beta}y_\beta + 6\,(\delta_3^2\sin\gamma g_1^\alpha -$$

$$- \delta_1^2\cos\gamma g_2^\alpha) - 2\,(R_1\sin\gamma - R_2\cos\gamma)\,m^\alpha\Big]$$

$$Q_{n+\alpha}^{(g)} + Q_{n+\alpha}^{*(g)} \approx -\frac{\mu}{r^2_{\bar{O}C}}\Big[2m^\alpha\delta_2 + \frac{1}{L}\sum_{\beta=1}^{n} m_*^{\alpha\beta}(\sin\gamma\delta_1 x_{n+\beta} - \cos\gamma\delta_3 y_{n+\beta})\Big] +$$

$$+ 3\frac{\mu}{r^3_{\bar{O}C}}\sum_{\beta=1}^{n}\Big\{-(\delta_1^2\sin\gamma\, d_2^{\alpha\beta} + \delta_3^2 d_1^{\alpha\beta}\cos\gamma)\,x_{n+\beta} + \delta_1\delta_3\,(\cos\gamma\, d_2^{\alpha\beta} +$$

$$+ \sin\gamma\, d_1^{\alpha\beta})\,y_{n+\beta} + A^{\alpha\beta}\delta_2\Big[\delta_1\cos\gamma y_\beta + \delta_3\sin\gamma x_\beta + \delta_2 x_{n+\beta}\Big]\Big\} -$$

$$-\frac{\mu}{r^3_{\bar{O}C}}\Big[\sum_{\beta=1}^{n} Q_{22}^{\alpha\beta}x_{n+\beta} + 6\delta_1\delta_2 g_2^\alpha\Big]$$

$$Q_{n+\alpha}^{(g)} - Q_{n+\alpha}^{*(g)} \approx -\frac{\mu}{r^3_{\bar{O}C}L}\sum_{\beta=1}^{n} m_*^{\alpha\beta}(\sin\gamma\delta_1 y_{n+\beta} - \cos\gamma\delta_3 x_{n+\beta}) +$$

$$+ 3 \frac{\mu}{r_{\bar{O}C}^3} \sum_{\beta=1}^{n} \Big\{ -(\delta_1^2 \sin\gamma\, d_2^{\alpha\beta} + \delta_3^2 \cos\gamma\, d_1^{\alpha\beta})\, y_{n+\beta} + \delta_1\delta_3 (d_2^{\alpha\beta} \cos\gamma + $$

$$+ d_1^{\alpha\beta} \sin\gamma)\, x_{n+\beta} + A^{\alpha\beta}\delta_2 [\delta_1 \cos\gamma x_\beta + \delta_3 \sin\gamma y_\beta + \delta_2 y_{n+\beta}]\Big\} -$$

$$- \frac{\mu}{r_{\bar{O}C}^3} \Big[\sum_{\beta=1}^{n} Q_{22}^{\alpha\beta} y_{n+\beta} - 6\delta_2\delta_3 g_1^\alpha \Big]$$

It should be noted that in Equations (2.11), (2.13), and (2.15) the inertia effects due to the translational motion of the body of the satellite are equilibrated by the components of the moments and generalized forces due to the action of the gravitational forces, which depend on the displacement of the center of inertia of the structure of the satellite, caused by its deformation.

The analytic determination of the corresponding generalized forces due to all the other external forces (e.g., aerodynamic and magnetic forces, etc.) acting on satellite of complex configuration, requires specific knowledge of their form and structure, and is not treated in the present article.

Let us turn now to the determination of the generalized forces F_α due to the internal reactions of the deformed stabilizers. Since the generalized coordinates q_α introduced above are measured from the natural state of the body, the expression for the potential energy of the elastic forces can be represented as a quadratic form in the variables [4], and for this problem is determined by the expression:

$$\Pi = \frac{1}{2} \sum_{\alpha=1}^{n} \sum_{\beta=1}^{n} c^{\alpha\beta} (q_\alpha q_\beta + q_{n+\alpha} q_{n+\beta} + q_\alpha^* q_\beta^* + q_{n+\alpha}^* q_{n+\beta}^*) \tag{2.24}$$

In (2.24)

$$c^{\alpha\beta} = \frac{EJ}{L^3} \int_0^1 \varphi_\alpha''(s)\, \varphi_\beta''(s)\, ds \tag{2.25}$$

where E is the modulus of elasticity of the material of the bars and J is the moment of inertia of transverse cross-sections of the stabilizers.

Then, in (2.15), the components of the generalized forces due to the elastic reactions are equal

$$\begin{aligned} F_\alpha + F_\alpha^* &= -\sum_{\beta=1}^{n} c^{\alpha\beta} x_\beta, \qquad & F_{n+\alpha} + F_{n+\alpha}^* &= -\sum_{\beta=1}^{n} c^{\alpha\beta} x_{n+\beta} \\ F_\alpha - F_\alpha^* &= -\sum_{\beta=1}^{n} c^{\alpha\beta} y_\beta, \qquad & F_{n+\alpha} - F_{n+\alpha}^* &= -\sum_{\beta=1}^{n} c^{\alpha\beta} y_{n+\beta} \end{aligned} \tag{2.26}$$

where the coefficients $c^{\alpha\beta}$ can be calculated if the modes of vibration of the stabilizers are known.

The terms Φ_α in (2.20) are very important, since they determine the character of the damping of the free vibrations of the stabilizers. The generalized forces Φ_α are determined from the internal damping in the material of the stabilizers as they deform and by the structural damping. It is possible to use the basic hypotheses which are usually employed in the description of internal damping arising in the vibrations of elastic systems [6], in order to determine these forces analytically. However, it is easier to make use of experimental data on the determination of the decrements of the vibrations, if these are available.

The projections of the vector $\boldsymbol{\omega}$ which occur in the system of equations of motion of the satellite are easy to determine if the following additional coordinate systems are introduced:

1. An equatorial coordinate system $O\xi_1\xi_2\xi_3$ with origin at the center of the Earth O, the axes $O\xi_1\xi_2$ lying in the Earth's equatorial plane, and $O\xi_3$ being directed along the axis of rotation of the Earth towards its North Pole (Fig. 3).

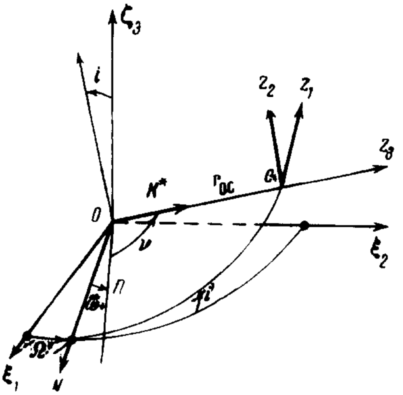

FIG. 3

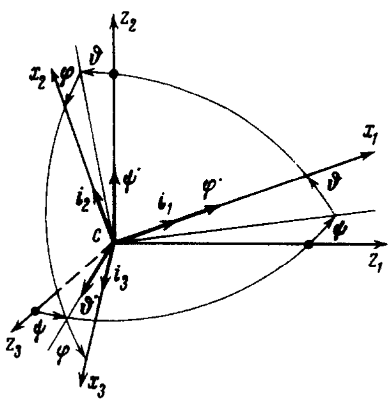

FIG. 4

2. An orbital system of coordinates $Cz_1z_2z_3$, which moves with the center of inertia C of the satellite and is formed by the radius vector r_{OC}, Cz_1 perpendicular to the radius vector, lying in the plane of the orbit and positive in the direction of motion of the satellite, and by the binormal to the orbit Cz_2 (Fig. 3).

3. Some auxiliary axes $Cx_1{}^*x_2{}^*x_3{}^*$ (moving with the center of inertia of the satellite) which coincide with $Cx_1x_2x_3$ axes of the structure when the axis of symmetry x_1 of the body is oriented in flight in the direction of the axis Cz_1, and which are rotated with respect to the $Cx_1x_2x_3$, axes when the axis of symmetry of the body x_1 is oriented in the direction of the radius vector r_{OC} (the axis Cz_3) or in the opposite direction (Fig. 4). Depending on its structural arrangement and purpose, either orientation is possible for a gravitational satellite. If the orientation of the axes $Cx_1{}^*x_2{}^*x_3{}^*$ with respect to the orbital axes is determined by the three angles: ψ - the pitch; θ - the jaw; and φ - the roll (Fig. 4), and if the elements of the transformation matrix between the axes $Cx_1{}^*x_2{}^*x_3{}^*$ and $Cz_1z_2z_3$ and the direction cosines between the axes $Cx_1{}^*x_2{}^*x_3{}^*$ and $Cx_1x_2x_3$ are given by the tables

	z_1	z_2	z_3
$x_1{}^*$	α_{11}	α_{12}	α_{13}
$x_2{}^*$	α_{21}	α_{22}	α_{23}
$x_3{}^*$	α_{31}	α_{32}	α_{33}

	x_1	x_2	x_3
$x_1{}^*$	a_{11}	a_{12}	a_{13}
$x_2{}^*$	a_{21}	a_{22}	a_{23}
$x_3{}^*$	a_{31}	a_{32}	a_{33}

(2.27)

then in Equations (2.11) to (2.13) and (2.15)

$$\begin{pmatrix}\omega_1\\ \omega_2\\ \omega_3\end{pmatrix} = \begin{pmatrix}a_{11} & a_{21} & a_{31}\\ a_{12} & a_{22} & a_{32}\\ a_{32} & a_{23} & a_{33}\end{pmatrix}\begin{pmatrix}\omega_1^*\\ \omega_2^*\\ \omega_3^*\end{pmatrix}, \qquad \begin{aligned}\omega_1^* &= \varphi^{\cdot} + \psi^{\cdot}\sin\vartheta + \Omega_1^*\\ \omega_2^* &= \vartheta^{\cdot}\sin\varphi + \psi^{\cdot}\cos\vartheta\cos\varphi + \Omega_2^*\\ \omega_3^* &= \vartheta^{\cdot}\cos\varphi - \psi^{\cdot}\cos\vartheta\sin\varphi + \Omega_3^*\end{aligned}$$

$$\begin{pmatrix}\Omega_1^*\\ \Omega_2^*\\ \Omega_3^*\end{pmatrix} = \begin{pmatrix}\alpha_{11} & \alpha_{12} & \alpha_{13}\\ \alpha_{21} & \alpha_{22} & \alpha_{23}\\ \alpha_{31} & \alpha_{32} & \alpha_{33}\end{pmatrix}\begin{pmatrix}\Omega_{z1}\\ \Omega_{z2}\\ \Omega_{z3}\end{pmatrix}, \qquad \begin{aligned}\Omega_{z1} &= -\frac{di}{dt}\sin u + \Omega^{\cdot}\sin i\cos u\\ \Omega_{z2} &= \omega^{+\cdot} + \nu^{\cdot} + \Omega^{\cdot}\cos i\\ \Omega_{z3} &= \frac{di}{dt}\cos u + \Omega^{\cdot}\sin i\sin u\end{aligned} \tag{2.28}$$

Here the generally accepted notation is used: Ω is the longitude of the ascending node of the orbit of the satellite, i is the inclination of the orbit, $u = \omega^+ + \nu$ is the argument of the latitude, ω^+ is the longitude of the perigee, ν is the true anomaly of the satellite.

In Equations (2.17), (2.19), and (2.23) the quantities δ_i are determined as

$$\begin{pmatrix}\delta_1\\ \delta_2\\ \delta_3\end{pmatrix} = \begin{pmatrix}a_{11} & a_{21} & a_{31}\\ a_{12} & a_{22} & a_{32}\\ a_{13} & a_{23} & a_{33}\end{pmatrix}\begin{pmatrix}\alpha_{13}\\ \alpha_{23}\\ \alpha_{33}\end{pmatrix} \tag{2.30}$$

on the basis of Equations (2.27) and (2.28).

If a gravitational satellite is provided with a special guidance or damping system, the equations of motion (2.11) to (2.15) must be supplemented by suitable differential equations which describe the process of guidance and dissipation of energy.

In the present article equations of rotational motion are given in orbital coordinates, for a satellite provided with deformable gravitational stabilizers. The deformations of the rods are presumed to be small in comparison with their lengths and are determined by a denumerable set of generalized coordinates. The equations which have been presented make it possible to solve a number of interesting problems in the dynamics of gravitational satellites.

Equations (2.11) to (2.15) permit the investigation of the effect of the motions of the end masses of the stabilizers and of the distributed mass of the stabilizers themselves, on the dynamics of a satellite, if the particular modes of vibration of the rods are specified, and the possibility of non-ideal attachment of the stabilizers to the satellite is accounted for. The effect of deformations of the stabilizers occurring as a result of solar heating of the structure [7] on the dynamics of the satellite may also be investigated. It is possible to estimate the stability and accuracy of orientation which can be expected from a passive gravitational satellite with flexible rods under the effects of external forces on the satellite.

A class of problems of special interest deals with the stability of the operation of active damping and guidance systems on a gravitational satellite when the stabilizers deform. The investigation of such problems is simplified considerably, if small vibrations of the satellite are examined in orbital coordinates and, if consideration of the deformations of the rods is limited to only the lowest modes of vibration. For instance in [8] an investigation is carried out of plane vibrations of a gravitational satellite with flexible stabilizers under the reactive moment of a preliminary damping system.

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